FES98 : A new global tide finite element solution independent of altimetry

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Abstract. A new version of the FES hydrodynamic finite element tide model combined with a revised data assimilation procedure is now available. The CEFMO hydrodynamic model previously used to obtain the solution FES94.1 was improved and the assimilation method based on the representer approach was revised. A careful selection of in situ tide gauge data from different data banks allowed us to build a collection of about 700 data values for each of the eight computed waves (M_2, S_2, N_2, K_2, $2N_2$, K_1, O_1 and Q_1), which were assimilated. Eighteen other constituents of the tide spectrum were derived by admittance and three long period waves were added. Referred to as FES98, these new solutions are available on a 0.25° x 0.25° gridded version of the full finite element solutions. This paper presents these new FES solutions and proposes a brief analysis of the improvements of the new approach with respect to the former FES94.1 and FES95.2.1 solutions.

1. Introduction

Most newly available tide models result from the analyses of satellite altimeter data (TOPEX/Poseidon and ERS1/2). They have shown major improvements with respect to previous global ocean tide models and have reached a very high level of accuracy, mainly in the deep oceans (at the centimeter level[*Shum et al.*, 1997]). However, the discrepancies are larger in shallow water (continental shelf and along coastlines) because of the complex and specific tidal characteristics. The wavelengths are shorter, and regional amplifications, often due to local resonance, result in sharp gradients which were hardly modelized by previous models. In addition nonlinearities occur in these areas, which lead to more complex tidal spectra than over the deep oceans.

The analyses of satellite data are very accurate in deep ocean areas which explains the quality of the recent tide models in these areas. But near coastlines these analyses have not been so much improved because of the technical problems occurring the treatment of the satellite measurements such as troposheric and ionospheric correction. Also the altimeter spatial sampling is poor.

On the other hand, tide gauges are very sparse in the deep ocean (a few bottom pressure gauge data values are available), but are numerous in shallow water. Moreover, tide gauge measurements are very accurate because of the short sampling period (an hour in general) and the quality of the harmonic analysis used to compute the tidal components. Hence, tide gauges provide an accurate information of the tidal phenomenon which occurs in shallow water compared to tidal data extracted from satellite altimetry, particularly limited by aliasing problems due to limited temporal (about 10 days for T/P) and spatial sampling.

So, the aim of our work was to use this tide gauge information to compute new tide finite element solutions (FES98) to produce global solutions improved mainly in shallow water and along the world coastlines. Besides, these new solutions are independent of altimetry.

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2. Methodology

2.1 Hydrodynamic model

The CEFMO hydrodynamic model previously used to obtain the solution FES94.1 [*Le Provost et al.*, 1994] was improved. Previously, the computation was realized by dividing the global ocean into sub-domains, which leads to the necessity to specify open boundary conditions. For FES94.1, tide gauge data were used to specify these conditions on each open side of the finite element meshes. The new formulation of the hydrodynamic model allows to compute the solution on the global scale. Hence, no more undesirable boundary effects are occurring [*Lyard*, 1999]. The new hydrodynamic solutions are totally free from in situ data : they are "purely hydrodynamic". These "free" solutions are forced only by the astronomical potential and secondary effects of loading and self-attraction provided by O. Francis (private communication) computed from the CSR3.0 solution [*Eanes and Bettadpur*, 1996]). They were computed for the eight main components of the tidal spectrum (M_2, S_2, N_2, K_2, $2N_2$, K_1, O_1 and Q_1). However, as the formulation of the hydrodynamic equations and the input parameters (such as bathymetry, loading and self attraction potential, friction...) are not exact, these "free" solutions are not as accurate as FES94.1 and FES95.2.1 [*Le Provost et al.*, 1998]. FES95.2.1 is an hydrodynamic solution improved by assimilating a satellite altimeter derived data set. Differences between the "free" solutions (FES98.free) and selected databases of measured data for M_2 and K_1 are shown in Table 1 (explanations of the comparison methods are given below).

2.2 Assimilation scheme

As mentioned above, the "free" hydrodynamic solutions lack accuracy. The use of further information from tide gauge analyses, according to an assimilation procedure, allows to reduce the errors between the "free" hydrodynamic solutions and the in situ measurements. The assimilation method is based on the representer approach [*Bennett*, 1992; *Egbert et al.*, 1994] and was formulated by *Lyard* [1999]. A representer is generated for each assimilated data point. It is representative of an instantaneous perturbation at the lo-

Table 1. RMS and RSS (in centimeters) of the comparisons between FES solutions and different databases.

Database	Solution	M_2	S_2	N_2	K_2[a]	$2N_2$[a]	K_1	O_1	Q_1	RSS
ST95	Data RMS	33.55	12.81	6.79	3.32	1.01	11.10	7.66	1.61	-
	Data number	95	95	91	90	90	95	95	87	-
	FES98.free	13.15	-	-	-	-	2.37	-	-	13.36
	FES94.1	2.85	1.57	0.91	0.48	*0.29*	1.19	1.09	0.28	3.80
	FES95.2.1	1.74	1.12	0.89	0.48	*0.29*	1.17	1.05	0.28	2.82
	FES98	*1.51*	*0.86*	*0.80*	*0.35*	0.31	*1.00*	*0.93*	*0.25*	*2.41*
ST727	Data RMS	57.78	22.04	12.18	6.54	1.84	13.48	9.74	2.01	-
	Data number	727	725	617	616	263	724	720	402	-
	FES98.free	25.86	-	-	-	-	6.28	-	-	26.61
	FES94.1	14.69	6.84	4.32	2.54	1.18	5.50	4.37	1.09	18.43
	FES95.2.1	22.02	18.82	5.12	2.54	1.18	7.08	4.54	*1.08*	30.74
	FES98	*10.99*	*5.56*	*3.66*	*2.24*	*1.09*	*4.71*	*3.56*	1.11	*14.40*
TOPEX	Data RMS	25.85	9.71	5.59	2.83	-	10.00	7.20	1.70	-
	Data number	5313	5313	5313	5313	-	5313	5313	5313	-
	FES98.free	13.25	-	-	-	-	2.77	-	-	13.54
	FES94.1	3.32	1.62	1.01	0.99	-	1.80	1.63	0.83	4.72
	FES95.2.1[b]	*1.33*	*0.82*	*0.75*	*0.97*	-	*1.11*	*0.87*	0.83	*2.57*
	FES98	2.51	1.19	0.82	0.98	-	1.61	1.14	*0.83*	3.73

[a]The FES95.2.1' K_2 and $2N_2$ solutions are the one of FES94.1

[b]The FES95.2.1 model used is masked in some areas which are affected by local resonance problems : 5239 data values were used in the comparisons.

The line called 'Data RMS' (respectively 'Data number') presents the RMS of the different tidal measurements from each of the three considered databases (respectively the number of tide gauges used in the comparisons) for each of the eight tidal components. For each column and each database, the italic number is the best RMS (cf. eq.1) of a FES solution compared to a database for a considered tidal component. The italic results indicate the best RMS for a wave.

cation of the assimilated data which propagates all over the world ocean discretized on a finite element mesh. Hence, the tidal hydrodynamic information is intrinsically contained within the representer. Its dimensions are those of the tidal elevation field. The whole assimilated solution is the sum of the "free" solution plus a weighted linear combination of the representers. The weights are determined by the confidence values which are given to each assimilated data value through the minimisation of a cost function.

3. Assimilated data

3.1 Selection of tide gauge datasets

Using tide gauge data is the aim of the present work, to improve the model solutions in shallow water areas independently of altimetry. Several databases of tidal spectra harmonic decomposition are available and cover many areas of the world ocean. The WOCE database [*Ponchaut et al.*, 1999] provides 155 accurate data mainly in deep ocean and island areas of the Pacific Ocean. The IAPSO [*Smithson*, 1992] set supplies 353 accurate data mainly in the deep ocean areas of the North Atlantic and the North Pacific. The IHO [*IHO*, 1979] database provides 3985 data with a wide range of accuracy along coastlines and on islands. However, they are on the whole very sparse in southern oceans. A big effort was done to filter the good from the bad components of the three databases (in particular the IHO dataset) as no selection was done so far for tide gauge assimilation purposes. A part of the selected values was used in assimilation, the other part for quality control. A careful selection allowed us to extract smaller datasets representative of the physics of tides along coastlines and in deep ocean areas, for each of the eight tidal components. Criteria based on tide gauge locations were applied to exclude nearby data or those which measure local phenomena such as resonance and extreme nonlinear effects. If measurement records are less than 28 days the data value was rejected as a poor analysis of the tidal spectrum. Consequently 763 values were selected for M_2 (cf. Fig. 1), 733 for S_2, 623 for N_2, 636 for K_2, 232 for $2N_2$, 877 for K_1, 881 for O_1 and 634 for Q_1. The number of selected tide gauges differs for each component. Indeed, each tide gauge harmonic analysis in the IHO dataset does not provide the same number of waves, depending on the person who performed the harmonic analysis, the type of algorithm used and the quality of the hourly time series.

3.2 Assimilation procedure

The representer algorithm requires to give a confidence (inverse of error) on each of the assimilated data and the model. A large confidence on a data value will magnify its influence whereas a small one will smooth it. Setting confidence on data values is a key-element of the assimilation scheme. As the tide gauge data used in the assimilation are not provided with error bars for each analysis, we set empirically the error for each data value. These errors depend on the quality of the harmonic analysis of the hourly sea surface elevation time series. Various parameters influenced these errors. So, thanks to our experience on data analyses, we classified the whole tide gauge dataset according to the localization from the land, the depth and the origin of the tide gauge. This led to the definition of four classes: deep ocean, shelf, island, coastal. In the deep ocean, interferences between waves are very small and the tidal signal can be considered as being composed of several main components: the analysis is expected to be good and we set the

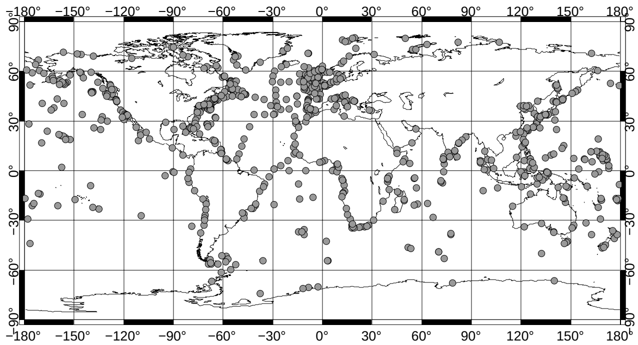

Figure 1. Locations of the tide gauges used in M_2 assimilation

error bar to 0.25 cm. In shallow water, along coastlines and near islands, it is not the case. As the analyses are expected to be less accurate, we set in these areas the error bar to 2.5 cm. To make a transition from shallow to deep waters, we introduced another class, the shelf one, with an error bar of 1 cm.

Due to a general inverse approach, we need to set the model error via the model forcing error. So, in this paper, we assumed that the forcing will have uniform variance and gaussian-shape spatial covariance.

4. Comparisons of the FES solutions

So as to evaluate the quality of the new FES98 solutions, comparisons are made with different databases of various origins, by using the root mean square (RMS) introduced by *Andersen et al.* [1995]:

$$RMS^2_{m-d} = \frac{1}{N}\sum_{N}\frac{1}{2}[(H_m cos G_m - H_d cos G_d)^2 + (H_m sin G_m - H_d sin G_d)^2] \qquad (1)$$

where the subscript m is for the model and d for data (tide gauge or altimetry). N is number of compared points. H_d (respectively H_m) is the data (respectively model) amplitude. G_d (respectively H_m) is the data (respectively model) Greenwich phase lag. To compare the eight computed waves a root sum square (RSS) is also calculated for each solutions:

$$RSS^2 = \sum_{waves} RMS^2 \qquad (2)$$

4.1 Comparisons to tide gauges in the deep ocean

The FES solutions are compared to the database of 95 tide gauges called ST95 [*Andersen et al.*, 1995]. This database was built to evaluate the quality of tidal models over the deep ocean. Table 1 highlights the improvements of FES98 compared to the two other FES solutions for the eight computed waves (except for $2N_2$, for which assimilated data are too less numerous). The accuracy now reaches 1.5 cm for M_2, and a RSS of 2.41 cm.

4.2 Comparisons to tide gauges along world coastlines

A nearly independent dataset (ST727, [*Lefèvre et al.*, 1999]) was selected for comparisons. Among this dataset 53 data are also found in the above introduced assimilation dataset, because of the uniqueness of their location. This dataset provides a coastal database that supplies tidal characteristics along the world coastlines. The comparisons of the FES solutions to ST727 Table 1 clearly show that the tidal information brought by the coastal tide gauges assimilated into FES98 improves the quality of the solutions along the coasts. Compared to the altimetric solutions FES95.2.1, the improvement of FES98 is by a factor 2. The RSS is equal to 14.40 cm, 4.03 cm below the one of FES94.1.

4.3 Comparisons to satellite altimetry

However, some of the tide gauge data are both used in the comparisons and the assimilation: for the M_2 wave solution, we used 58 (respectively 53) tide gauges of ST95 (respectively ST727) for the assimilation. So as to compare the non-altimetric FES98 solutions to a completely independent database, an altimetric satellite database was used, supplied by *Schrama* (personal communication) providing a harmonic crossover analysis of the tide spectrum on more than 5000 points on the world ocean. FES98 solutions are also improved over FES94.1 for M_2 (respectively K1) by 25% (11% respectively). The altimetric FES95.2.1 solution remains the best one due to the comparison dataset which is also altimetric.

5. Conclusion

A new version of our hydrodynamic finite element tide model combined with a revised data assimilation procedure is now available. The improvements of the CEFMO hydrodynamic model allow to compute tidal solutions at the global scale. "Free" solutions, forced by the astronomical potential and secondary effects of loading and self-attraction, were computed for M_2, S_2, N_2, K_2, $2N_2$, K_1, O_1 and Q_1. The assimilation method based on the representer approach was revised, and the anomalous resonances observed over some specific coastal areas in FES95.2.1 were eliminated. For each of the eight computed waves, a collection of (763, 733, 623, 636, 232, 877, 881, 643) data respectively for (M_2, S_2, N_2, K_2, $2N_2$, K_1, O_1, Q_1) was built by carefully extracting in situ data from three tidal data banks (IAPSO, WOCE and BHI). These datasets were assimilated into the "free" hydrodynamic solutions. These solutions, independent of altimetric measurements, are more accurate than the FES94.1 hydrodynamic and FES95.2.1 altimetric tide solutions. To complement the FES98 tidal spectrum, eighteen other constituents (μ_2, ν_2, L_2, λ_2, T_2, ϵ_2, η_2, P_1, $2Q_1$, σ_1, ρ_1, M_1, ξ_1, π_1, ϕ_1, θ_1, J_1, OO_1) were deduced from splines or linear admittance as for FES95.2.1 solutions [*Le Provost et al.*, 1998]. Three long period waves were also added and were computed by *Lyard et al.* [1999] : M_f, M_m were computed thanks to the assimilation algorithm and Mt_m is hydrodynamic. These new FES98 solutions are available on a 0.25 $^\circ$ x 0.25 $^\circ$ gridded version of the full finite element solutions and can be supplied on request.

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